Rotational Motion

Angular Displacement, Velocity, Acceleration

Rotation w/constant angular acceleration

Linear vs. Angular Kinematics

Rotational Energy

Parallel Axis Thm.

Angular Displacement (θ)

- Angular = Rotational
- Measured in Radians
- 1 complete rotation = 2π radians

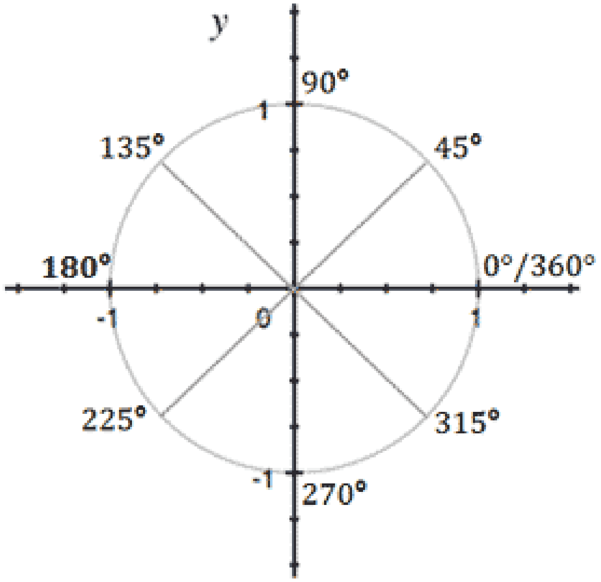


Figure 1: Unit circle measured in degrees.

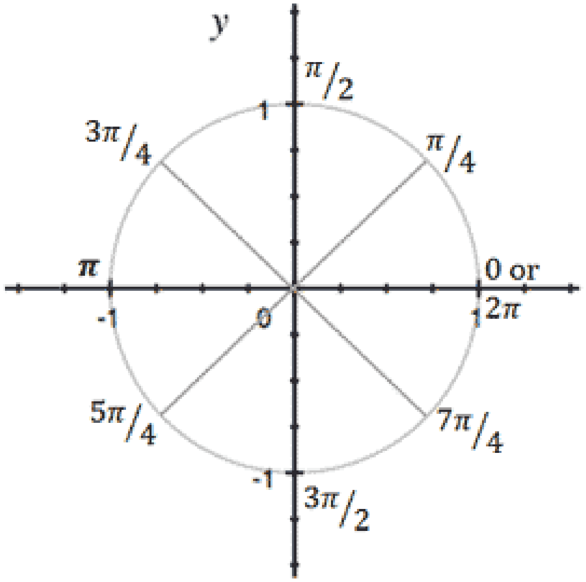
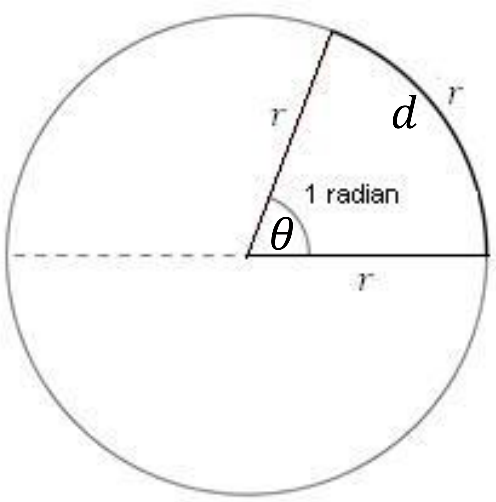


Figure 2: Unit circle measured in radians.

- Easy to convert from angular to translational
- $d\ = \theta r$



Examples

A dog (r = 0.12m) make 3 complete rotations as it rolls own a hill.

- What is the angular displacement of the dog?
- What is the linear displacement dog?

- A cat runs 7m on wheel with a radius of 0.75 m. How many rotations does the wheel make?

Angular Velocity = (ω)

- $v = \frac{d}{t} = \frac{\theta}{t} = {}^{rad}/_{sec}$
- rpm = rotations per minute
- $rpm * \frac{2\pi}{60} = \omega$
- $v = \omega r$

EXAMPLE

- Convert 45rpm to rad/s
- What is the translation Velocity on the outer edge of the record? (diameter = 30cm)

Angular Acceleration = (α)

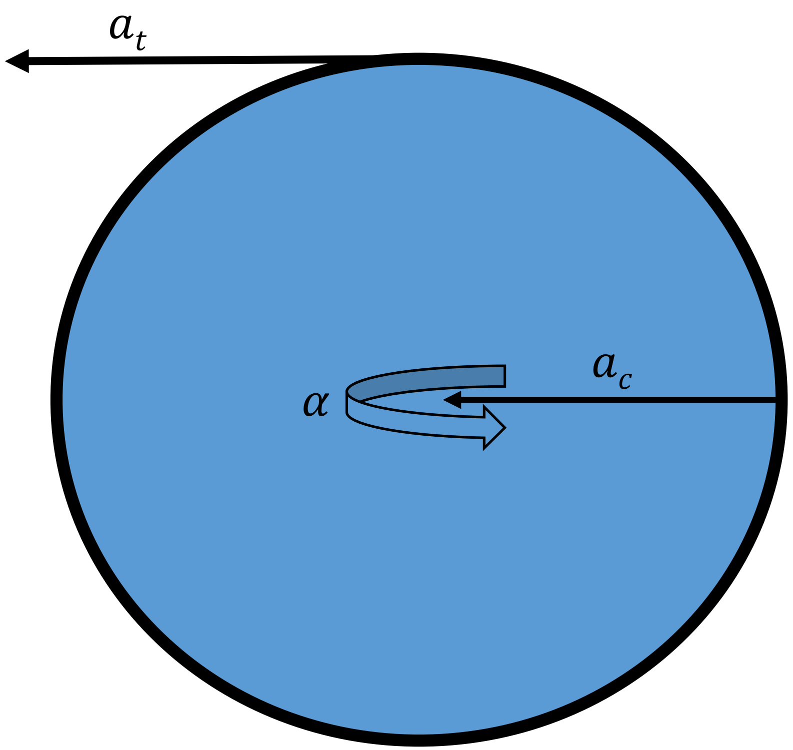
- $\alpha = \frac{\Delta\omega}{t}$

Translational Acceleration

- $a_t = \alpha r$

Centripetal Acceleration

- $a_c = \frac{v^2}{r} = \omega^2 r$



Example:

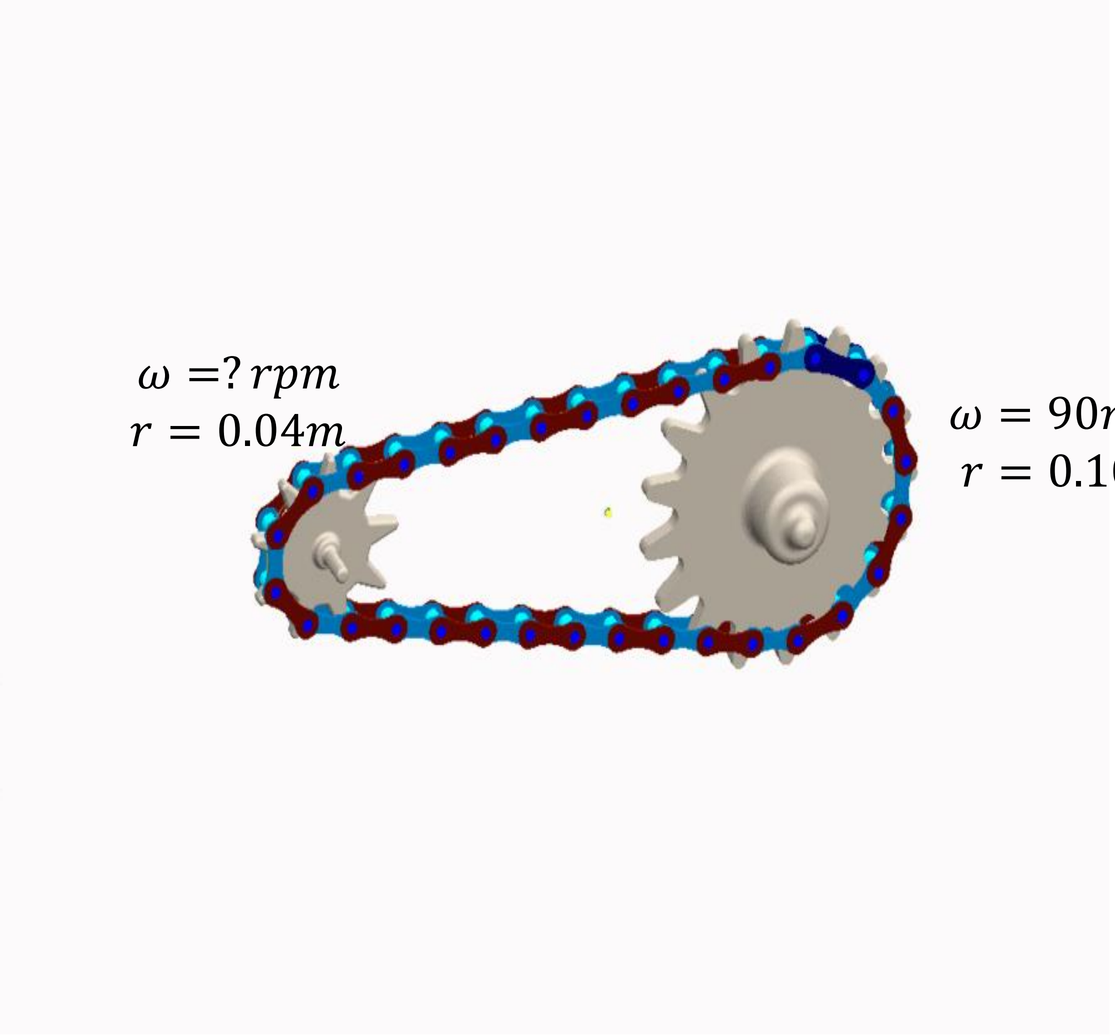
A Washing machine motor accelerates from 0 -200rpm in 5 seconds.

- What is the angular acceleration of the motor in rad/s^2?

- A small pony is in the machine (r = 0.30m), what is the translational acceleration of the pony as the motor accelerates?

- What is the centripetal acceleration of the pony?

Example:



Motion with Constant Acceleration

$v = \frac{d}{t}$ $\omega = \frac{\theta}{t}$	d=vt +d_i $\theta = \omega t + \theta_i$	$a = \frac{\Delta v}{t}$ $\alpha = \frac{\Delta \omega}{t}$	v_f =at + v_i $\omega = \alpha t + \omega_i$

$d = \frac{1}{2}at^2 + vit \ + di$ $\theta = \frac{1}{2}\alpha t^2 + \omega_i t \ + \theta_i$	$d = \frac{1}{2}(v_1 + v_2)t$ $\theta = \frac{1}{2}(\omega_1 + \omega_2)t$	${v_f}^2 = 2ad + vi^2$ ${\omega_f}^2 = 2\alpha\theta + {\omega_i}^2$

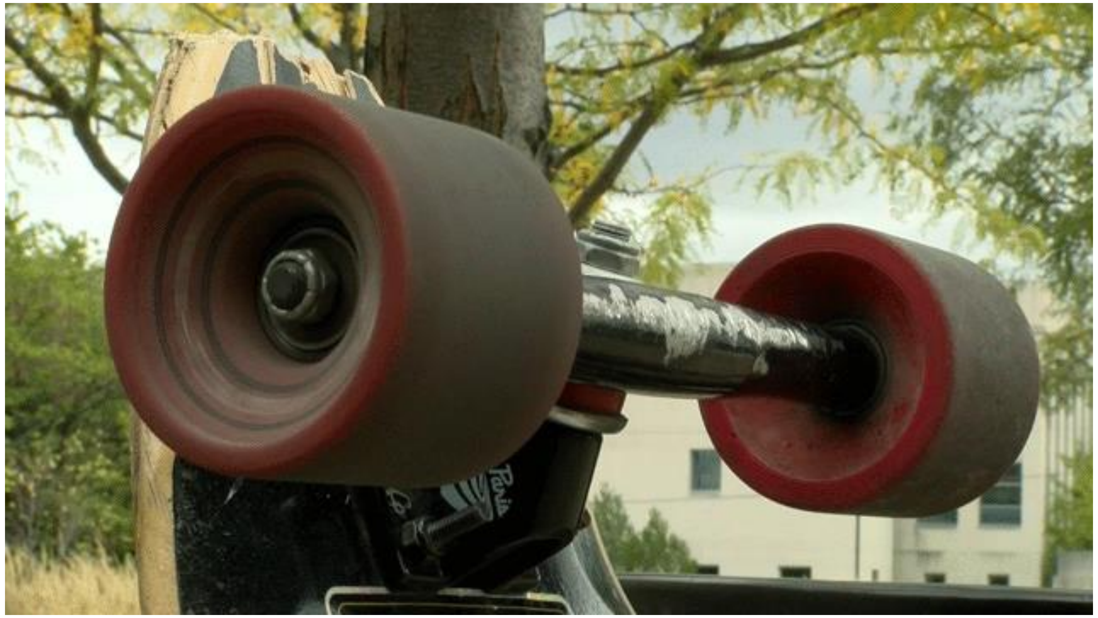
Example:

A biker accelerates from rest to 8 m/s in 5 seconds.

The radius of the bicycle wheels is 0.25 m.

- What is the angular acceleration of the wheels?
- How far does the biker travel?
- How many rotations does the wheel make?
- If he gets tired and comes to rest in 10m, what is the angular acceleration of the wheels?

Energy in Rotational Motion



Objects in motion have Kinetic Energy

Rotating Objects have Kinetic Energy

Energy of single particle:

$$KE = \frac{1}{2}mv^2 \rightarrow v = \omega r \rightarrow \frac{1}{2}m(\omega r)^2$$

$$KE = \frac{1}{2}(mr^2)\omega^2 \rightarrow mr^2 = I = moment\ of\ Inertia = \sum_i m_i {r_i}^2$$

$$KE = \frac{1}{2}I\omega^2$$

Moment of Inertia *(rotational mass)*

- Resistance to changes in Rotational Motion
- Depends in distribution of mass.

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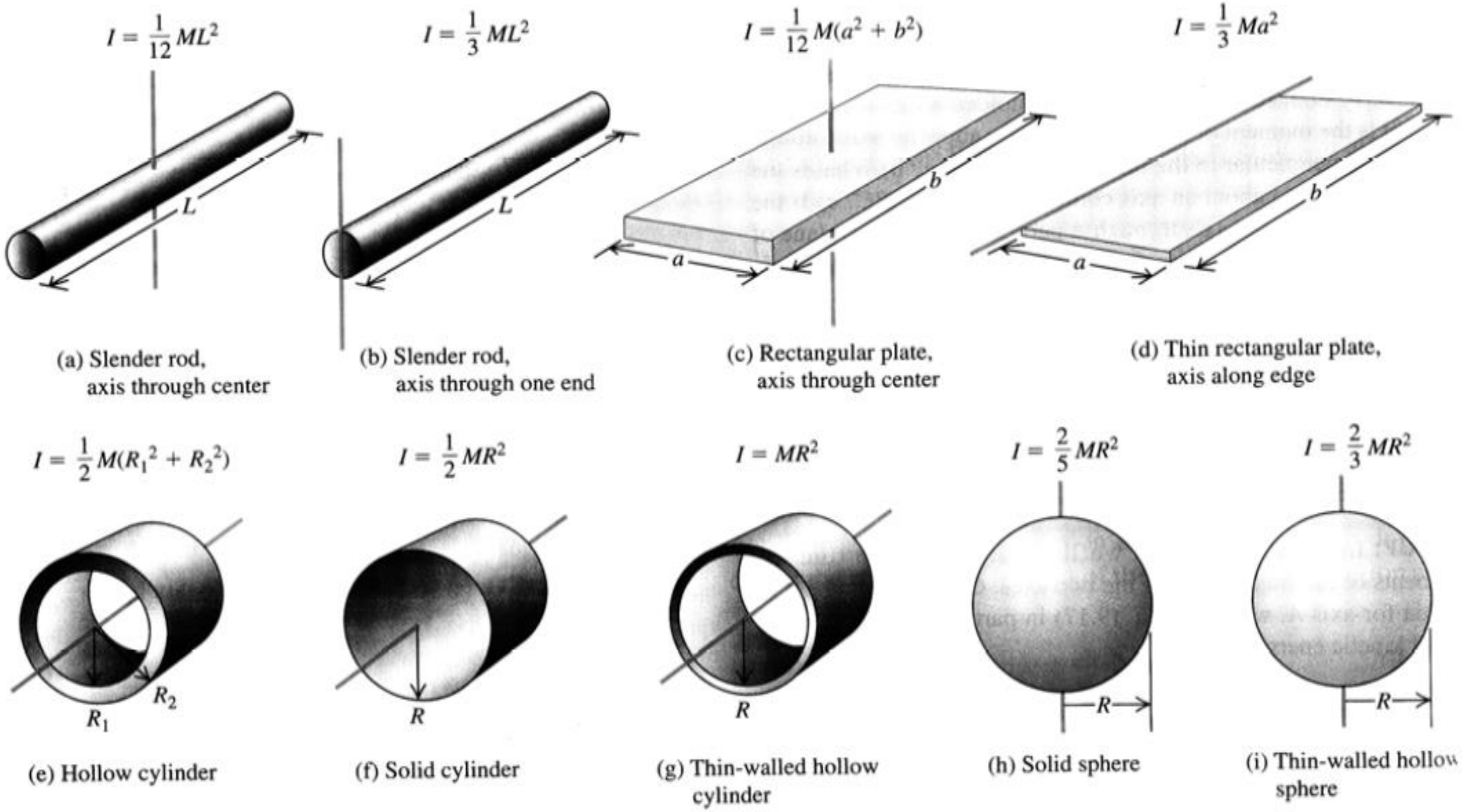


(a) Slender rod, axis through center

(b) Slender rod, axis through one end

(c) Rectangular plate, axis through center

(d) Thin rectangular plate, axis along edge

(e) Hollow cylinder

(f) Solid cylinder

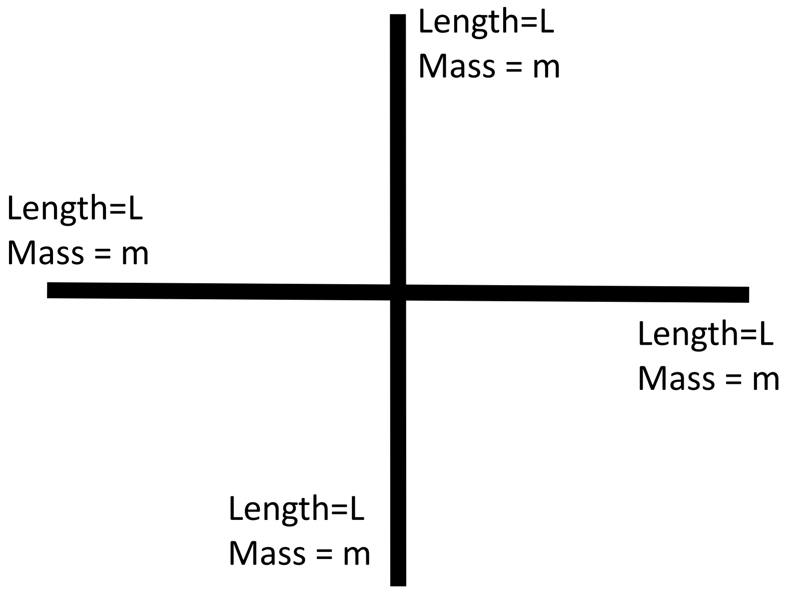
(g) Thin-walled hollow cylinder

(h) Solid sphere

(i) Thin-walled hollow sphere

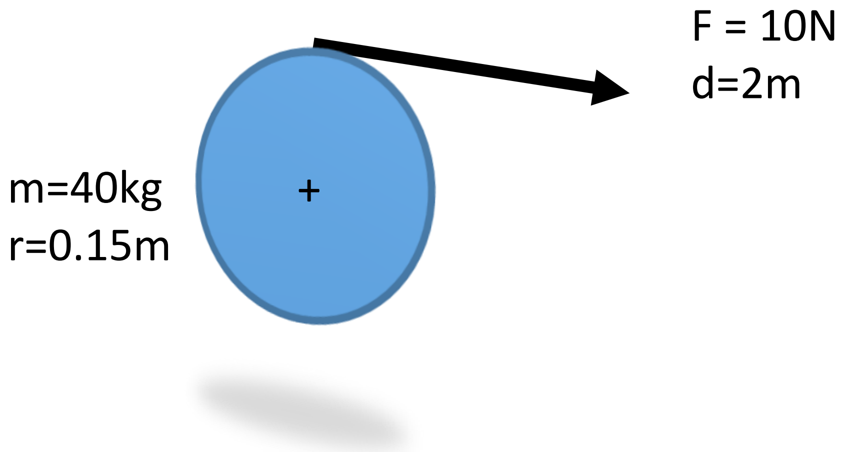
Moment of Inertia

determine the moment of Inertia around the center of the object below



Energy in Rotational Motion *(work energy thm.)*

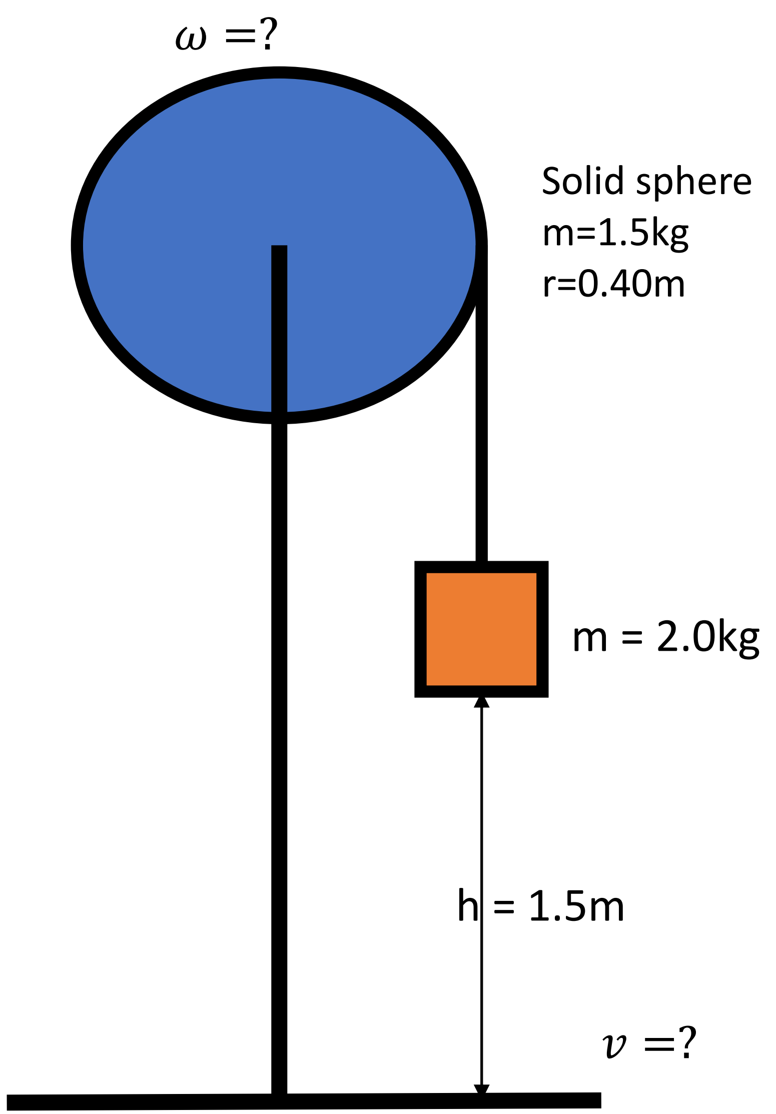
A force of 10N acts for 2.0m on 40 kg Solid Disc with a radius of 0.15m. What is the final angular speed of the wheel as it spins on its axis?



$$W = \Delta KE$$

$$I = \frac{1}{2}mr^2$$

Energy in Rotational Motion



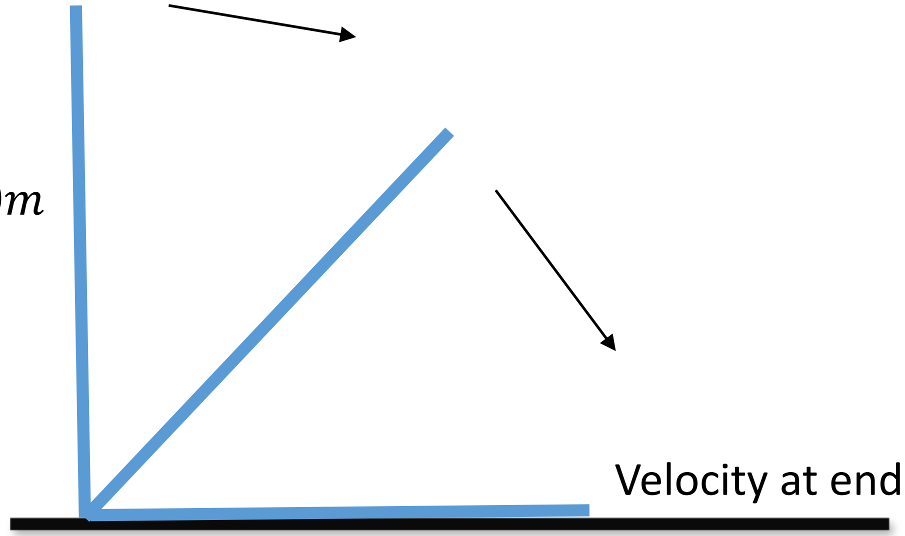
A 2kg block is tied to the outer edge of a 1.5kg sphere with a diameter of 0.80m. The block is allowed to fall 1.5m, causing the sphere to rotate.
What is the final speed of the wheel and the block?

$$\Delta GPE = KE_1 + KE_2$$

Energy in Rotational Motion

A meter stick standing on one end is allowed to fall,
What is the speed of the end of the meter stick?

$l = 1.0m$



Velocity at end of Stick?

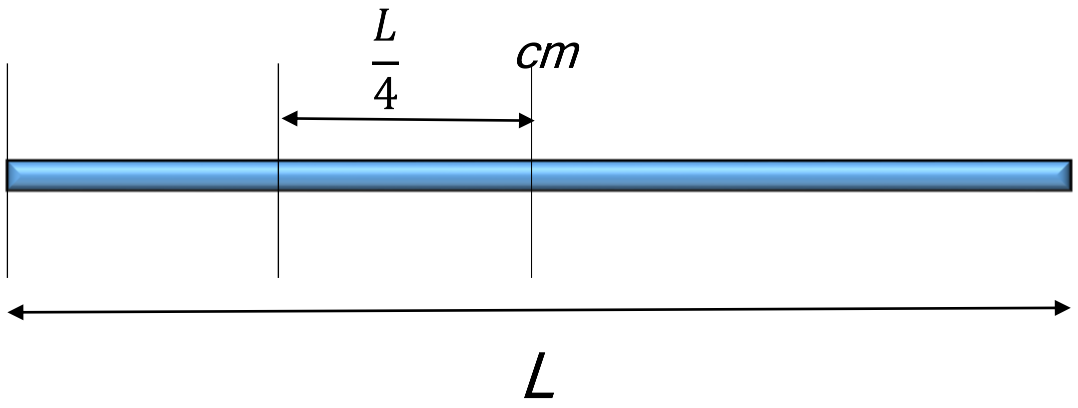
$$\Delta GPE = KE$$

Parallel Axis Thm.

- Determine the moment of Inertia for alternative axis of rotation.

$$I_p = Icm + Md^2$$

$I_{cm} = Moment\ of\ Inertia\ around\ Center\ of\ mass$
$d = distance\ from\ axis\ to\ center\ of\ mass$



Using the Parallel Axis Thm., determine the moment of Inertia around the two positions shown.

Parallel Axis Thm.

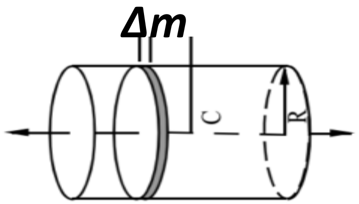
Using the Parallel Axis Thm, determine the moment of Inertia around the axis shown through the Hollow sphere.

Determine Moment of Inertia

Evaluate the moment of Inertia by modeling the object divideded into many small "volume elements" of mass $\boldsymbol{\Delta m}$

$\boldsymbol{I = \Sigma(mr^2) \rightrightarrows I = \int r^2\, dm}$

Challenge: What is $\boldsymbol{dm}$?

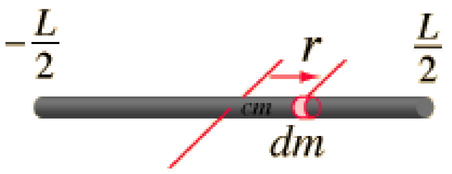


For 3D objects:

$\boldsymbol{dm}$ expressed in Volume Density

$$\rho = \frac{dm}{dV} \longrightarrow dm = \rho * dV$$

$$I = \int \rho r^2 * dV$$



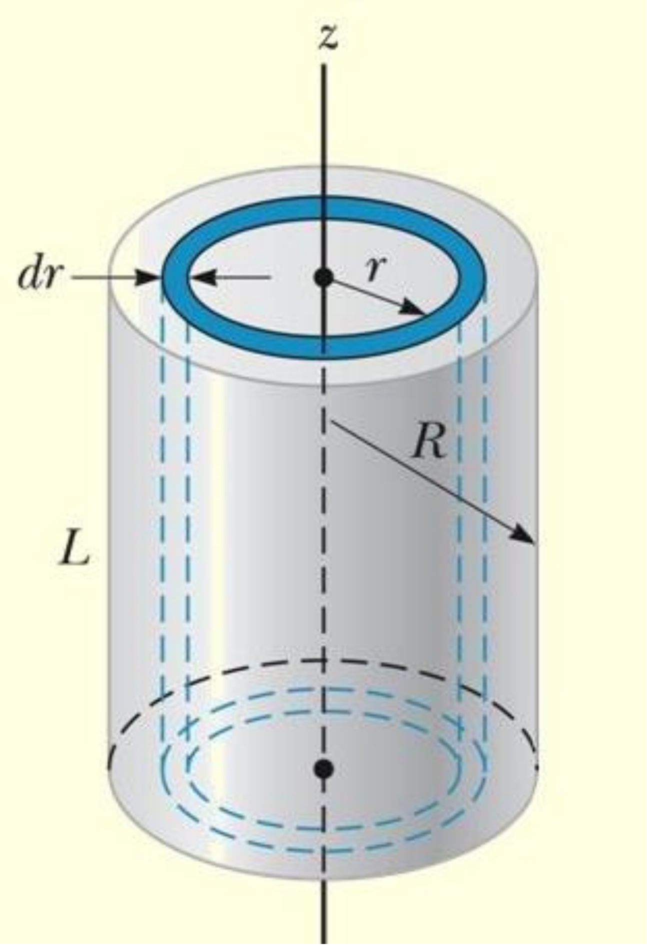
For 2D objects:

$\boldsymbol{dm}$ expressed in Linear Density

$$dm = \frac{M}{L}$$

$$I = \int r^2 * dm = \int r^2 * \frac{M}{L}$$

Determine Moment of Inertia



Uniform (solid) Cylinder

$$I = \int r^2 * dm \int \rho r^2 * dV$$

$$dm = \rho * dV$$

$$dV = Area \ * Ldr = (2\pi r)dr * L$$

$$dm = \rho(2\pi r)L$$

$$\rho = \frac{M}{V} = \frac{M}{L * \pi r^2}$$

Calculation:

$$I = \int r^2 * dm \int r^2 * \rho dV$$

$$I = \int r^2 * \rho(2\pi r)L$$

$$I = \rho 2\pi L \int r^3$$

$$I = \rho 2\pi L * \frac{r^4}{4}$$

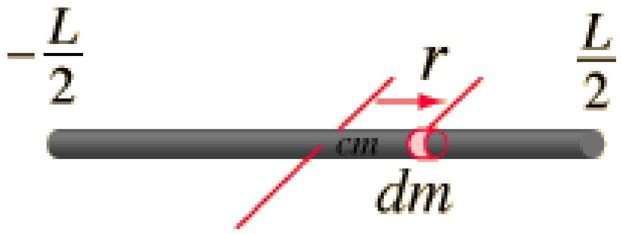
$$I = \frac{M}{L * \pi r^2} * 2\pi L * \frac{r^4}{4}$$

Simplify

$$I = \frac{1}{2} mr^2$$

Determine Moment of Inertia

Uniform Rod through the center *(2D object)*



$$I = \int r^2 * dm$$

$$dm = \frac{M}{L}$$

Calculation:

$$I = \int_{-L/2}^{L/2} r^2 * dm$$

$$I = \int_{-L/2}^{L/2} r^2 * \frac{M}{L} = \frac{M}{L} \int r^2$$

$$I = \frac{M}{L} \left[\frac{r^3}{3}\right]_{-L/2}^{L/2}$$

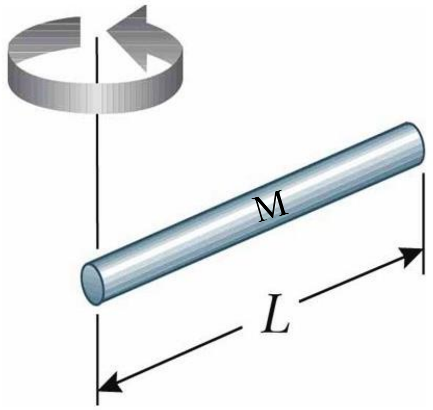
$$I = \frac{M}{3L} [r^3 - r^3]$$

Simplify:

$$I = \frac{1}{12} ML^2$$

Determine Moment of Inertia

Uniform Rod through one end *(2D object)*



$$I = \int r^2 * dm$$

$$dm = \frac{M}{L}$$

Calculation: *(you do it)*

$$I = \int_0^L r^2 * dm$$